A Generalized Correlation for Flashing Choked Flow of Initially Subcooled Liquid

J. C. Leung, M. A. Grolmes
Fauske & Associates, Inc.
Burr Ridge, IL 60521

This paper presents, for the first time, a generalized correlation for flashing choked flow of an initially subcooled liquid. The present approach is an extension of a previously published correlation for homogeneous equilibrium choked flow with two-phase (quality) inlet conditions (Leung, 1986). The model assumptions are:

1. Isentropic flow
2. Thermal equilibrium
3. Equal phasic velocities once saturation is reached

It should be noted that this model is only a limiting case, without consideration of nonequilibrium effects (Schrock et al., 1977). In this regard it gives a lower-bound estimate for the mass flow rate and should therefore serve as a useful reference model in many engineering applications.

Model Development

As usual, the critical mass velocity G is found by maximizing the following equation derived from the first and second laws

$$G = \left(2 \int_{P_o}^{P} - v dP\right)^{1/2} \Big/ v \tag{1}$$

For a subcooled inlet condition the above integral can be partitioned into two parts, namely the subcooled region and the saturated two-phase region, thus:

$$\frac{1}{2}(Gv)^2 = \underbrace{-\int_{P_o}^{P_s} v_{fo} dP}_{\text{I}} \underbrace{- \int_{P_s}^{P} v dP}_{\text{II}} \tag{2}$$

By treating the liquid as an incompressible fluid, the first integral on the righthand side becomes

$$\text{I} = v_{fo}(P_o - P_s) \tag{3}$$

In the saturated two-phase region, we employ the approximate equation of state in the form (Epstein et al., 1983)

$$\frac{v}{v_{fo}} = \omega\left(\frac{P_s}{P} - 1\right) + 1 \tag{4}$$

where ω is given by (Leung, 1986)

$$\omega = \frac{C_{fo} T_o P_s}{v_{fo}} \left(\frac{v_{fgo}}{h_{fgo}}\right)^2 \tag{5}$$

Here all physical properties are to be evaluated at the saturation line corresponding to the inlet stagnation temperature T_o. P_s is the saturation pressure. Upon substitution, the second integral becomes

$$\text{II} = v_{fo}\left[\omega P_s \ln\left(\frac{P_s}{P}\right) - (\omega - 1)(P_s - P)\right] \tag{6}$$

By defining

$$\eta \equiv P/P_o,\ \eta_s \equiv P_s/P_o$$

and

$$G^* = G/\sqrt{P_o/v_{fo}} = G/\sqrt{P_o \rho_{fo}}$$

Eq. 2 yields an expression for the normalized mass velocity as

$$G^* = \frac{\left\{2(1 - \eta_s) + 2\left[\omega\eta_s \ln\left(\frac{\eta_s}{\eta}\right) - (\omega - 1)(\eta_s - \eta)\right]\right\}^{1/2}}{\omega\left(\frac{\eta_s}{\eta} - 1\right) + 1} \tag{7}$$

Maximization is accomplished by setting the derivative dG/dP or $dG^*/d\eta$ to zero, thus yielding the following transcendental equation for the critical pressure ratio η_c

$$\frac{\left(\omega + \frac{1}{\omega} - 2\right)}{2\eta_s}\eta_c^2 - 2(\omega - 1)\eta_c + \omega\eta_s \ln\left(\frac{\eta_c}{\eta_s}\right) + \frac{3}{2}\omega\eta_s - 1 = 0 \quad (8)$$

and the associated critical mass velocity expression

$$G_c^* = \frac{\eta_c}{(\omega\eta_s)^{0.5}} \quad (9a)$$

or in dimensional form

$$G_c = \eta_c\left(\frac{P_o\rho_{fo}}{\omega\eta_s}\right)^{1/2} \quad (9b)$$

Equation 8 can be solved for η_c using Newton's iteration method. However, we also pursued an approximate solution by replacing the logarithmic term with a second-order Taylor series expansion. The resulting quadratic expression in η_c yields the following explicit solution

$$\eta_c = \eta_s\left(\frac{2\omega}{2\omega - 1}\right)\left[1 - \sqrt{1 - \frac{1}{\eta_s}\left(\frac{2\omega - 1}{2\omega}\right)}\right] \quad (10)$$

This expression suggests, however, that in order for a solution to exist the following inequality has to be safisfied

$$\eta_s \geq \frac{2\omega - 1}{2\omega} \quad (11)$$

This forms the transition criterion between two distinct regions—see Eq. 14 and discussion. In the low subcooling region where this inequality is satisfied, the fluid attains flashing prior to reaching the choked location. On the other hand, at high subcooling no vapor is formed until the choked location is reached. In this latter instance, the second integral in Eq. 2 drops out and one simply obtains the familiar Bernoulli equation

$$G_c^* = [2(1 - \eta_s)]^{0.5} \quad (12a)$$

or

$$G_c = [2\rho_{fo}(P_o - P_s)]^{0.5} \quad (12b)$$

and the critical pressure ratio as given by

$$\eta_c = P_s/P_o \quad (13)$$

In other words, the critical pressure is given by the saturation pressure corresponding to the stagnation temperature. In this high subcooling region, the present method gives essentially the same results as Henry's (1970) nonequilibrium subcooled model.

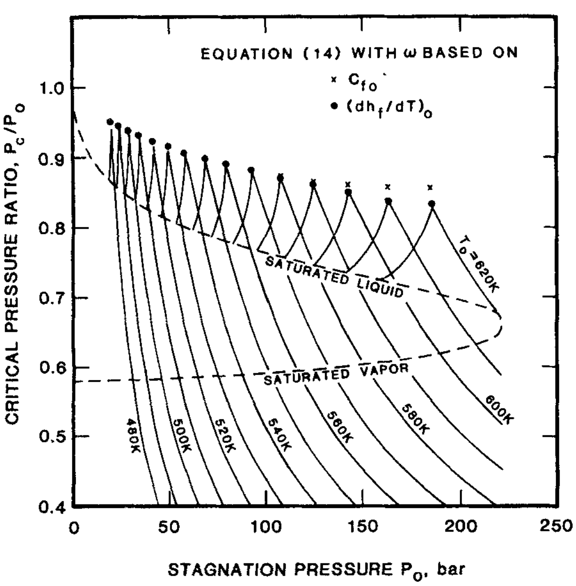

Figure 1. Comparison of suggested transition criterion with water data of Hall and Czapary (1980).
Adapted from Hall & Czapary Fig. B-3

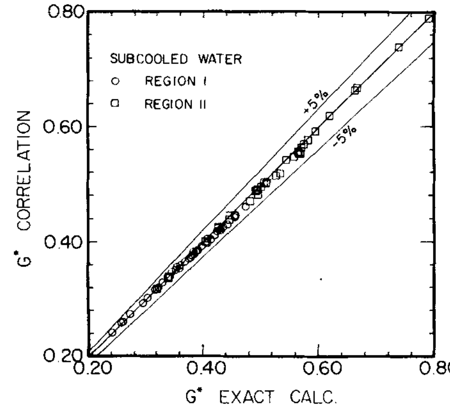

Figure 2. Normalized mass velocity.
Predictions, Eqs. 8 and 9, vs. exact calculations

Comparison of Model and Literature Data

Comparison is made between the present correlation and the very extensive tabulation for homogeneous equilibrium critical flow for water by Hall and Czapary (1980) in the entire sub-

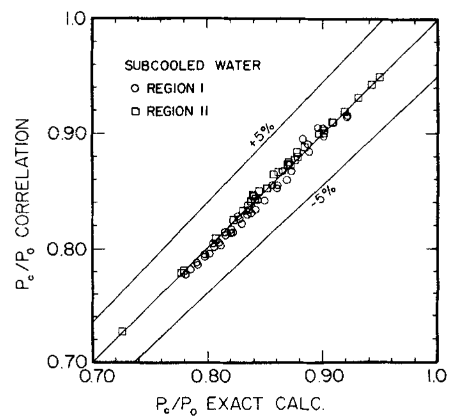

Figure 3. Critical pressure ratio.
Predictions, Eq. 8, vs. exact calculations

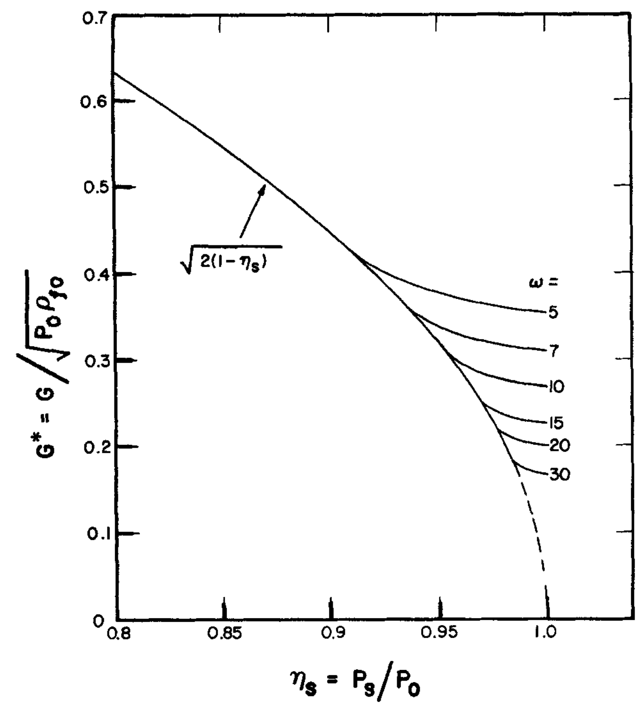

Figure 4. Design chart for normalized mass velocity.

cooled inlet regime. First, the transition critical pressure ratio as predicted by

$$\eta_{ct} = \frac{2\omega - 1}{2\omega} \tag{14}$$

is in excellent agreement with their computational results as shown in Figure 1. As the stagnation temperature approaches the critical temperature, the present correlation with ω evaluated based on the liquid specific heat at constant pressure, Eq. 5, begins to overpredict the transition critical pressure ratio. However, as was found in the previous work (Leung, 1986), the current prediction is significantly improved in the near-critical region by replacing C_{fo} with $(dh_f/dT)_o$.

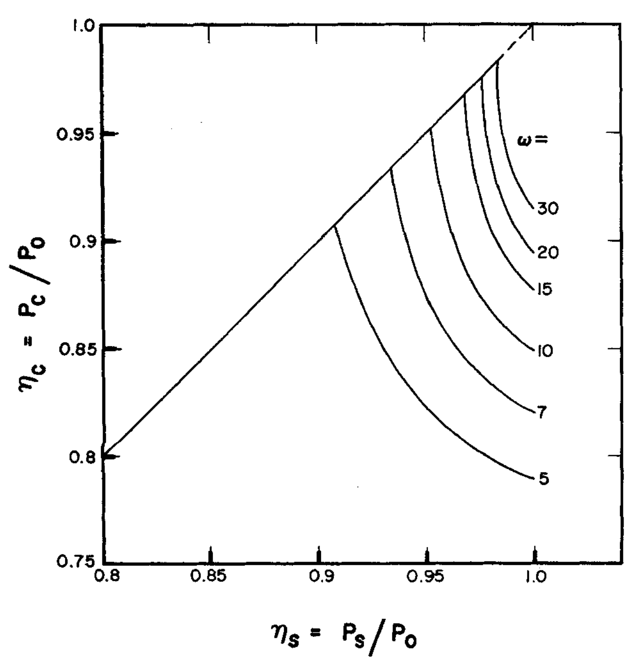

Figure 5. Design chart for critical pressure ratio.

Figures 2 and 3 are comparisons between the present calculational method and the computed results of Hall and Czapary in terms of the normalized critical mass velocity and the critical pressure ratio, respectively. As for the prediction, the transcendental equation for η_c, that is, Eq. 8, was employed in the low subcooling region (region I). All of the available low subcooling data of Hall and Czapary (T_o from 460 to 600 K in 10 K increments) were used, yielding a total of 43 points. In the high subcooling region (region II) three points were chosen from each temperature level, giving a total of 36 points. Both figures clearly illustrate the goodness of fit of the pesent scheme; the standard deviations σ for G^* and η_c were 1.3 and 0.6%, respectively.

In addition, the approximate expression for η_c, Eq. 10, for low subcooling yielded a σ of 2.4% in region I, and the corresponding G prediction resulted in a σ of 2.7%. In view of their simple analytical closed forms, Eqs. 9 and 10 should prove useful in most engineering calculations.

Design Charts

Finally, Figures 4 and 5 present design charts based on the present scheme. These charts illustrate the demarcation between the high and low subcooling regions. However their usefulness for design engineers lies in their ability to give quick answers. As an example, a Hall and Czapary table gives a G value of 23,940 kg/m²s and a η_c value of 0.838 for the following stagnation conditions:

$$T_o = 540\text{K} \qquad \rho_{go} = 26.9\ \text{kg/m}^3$$
$$P_o = 5.5\ \text{MPa} \qquad h_{fgo} = 1{,}622.9\ \text{kJ/kg}$$
$$P_s = 5.24\ \text{MPa} \qquad C_{fo} = 5{,}039\ \text{J/kg}\cdot\text{K}$$
$$\rho_{fo} = 773\ \text{kg/m}^3$$

Thus the saturation pressure ratio η_s is 0.953 and the correlating parameter ω, Eq. 5, is calculated to be 5.4. According to Figures 4 and 5, these two parameters determine the G^* and η_c values to be 0.365 and 0.83, respectively. The critical mass velocity $G^*/\sqrt{P_o\rho_{fo}}$ is hence 23,800 kg/m² · s. Thus both G and critical pressure ratio predictions are in good agreement with the "exact" calculation.

Notation

C_f = liquid specific heat at constant pressure
G = mass velocity or flux
G^* = normalized mass velocity
h_f = liquid specific enthalpy
h_{fg} = latent heat of vaporization
P = pressure
T = temperature
v = specific volume

Greek letters

ω = correlating parameter, Eq. 5
η = pressure ratio
ρ = density

Subscripts

c = critical
f = liquid phase
fg = difference between vapor and liquid phase property
o = stagnation condition
s = saturation

Literature Cited

Epstein, M., R. E. Henry, W. Midvidy, and R. E. Pauls, "One-Dimensional Modeling of Two-Phase Jet Expansion and Impingement," *Thermal Hydraulics of Nuclear Reactors II, 2nd Int. Topical Meet. Nuclear Reactor Thermal Hydraulics*, Santa Barbara, CA (Jan., 1983).

Hall, D. G., and L. S. Czapary, "Tables of Homogeneous Equilibrium Critical Flow Parameters for Water in SI Units," Rept. No. EGG-2056, EG&G Idaho, Inc. (Sept., 1980).

Henry, R. E., "The Two-Phase Critical Discharge of Initially Saturated or Subcooled Liquid," *Nucl. Sci. Eng.*, **41**, 336 (1970).

Leung, J. C., "A Generalized Correlation for One-Component Homogeneous Equilibrium Flashing Choked Flow," *AIChE J.*, **32**(10), 1743 (Oct., 1986).

Schrock, V. E., E. S. Starkman, and R. A. Brown, "Flashing Flow of Initially Subcooled Water in Convergent-Divergent Nozzles," *ASME J. Heat Trans.*, **99**, 263 (1977).

Manuscript received June 19, 1987 and revision received Sept. 17, 1987.